

Closing tonight: 2.8

Closing Fri: 3.1-2

Closing Mon: 3.3 (finish sooner!)

Exam 1 is Tuesday, Jan 31st in your normal quiz section. Covers 2.1-2.3, 2.5-2.8, 3.1-3.3.

- One 8.5 by 11 inch sheet of **handwritten** notes (front and back)
- A Ti-30x IIs calculator (this model only!)
- Pen or pencil (no red or green)
- No make-up exams.

All homework is fair game. Know the concepts well. Practice on old exams.

$$1. \frac{d}{dx}(c) = 0$$

$$2. \frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

$$3. \frac{d}{dx}(cf(x)) = cf'(x)$$

$$4. \frac{d}{dx}(x^n) = nx^{n-1}$$

$$5. \frac{d}{dx}(e^x) = e^x \text{ and } \frac{d}{dx}(a^x) = a^x \ln(a)$$

Entry Task:

Find the derivatives of

$$a) g(x) = \frac{x^3}{2} - \frac{3}{\sqrt{x^5}} + \frac{e^x}{10}$$

$$b) f(x) = \frac{20}{3}x^3 - \frac{7x^2}{2} - 6x + 90$$

c) Find all x at which $y = f(x)$ has a horizontal tangent.

Application Notes:

1. $f'(a)$ = “slope of tangent to $f(x)$ at a ”

2. *Tangent Line Equation:*

$$y = f'(a)(x - a) + f(a)$$

3. $-\frac{1}{f'(a)}$ = “slope of *normal* to $f(x)$ at a ”

4. *Normal Line Equation:*

$$y = -\frac{1}{f'(a)}(x - a) + f(a)$$

Example: Let $f(x) = x^2 + 3$.

a) Find $f'(x)$

b) Find the equations for the tangent and normal lines at $x = 2$.

c) Find all points on $y = f(x)$ at which the normal line would also pass through $(0,10)$

3.2 Product and Quotient Rules

$$6. \frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$$

$$7. \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

Examples: Find y'

$$a) y = x^4 e^x$$

$$b) y = \frac{2x^3}{x^2 + 4}$$

6. Product Rule Proof:

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h} \\ &= \lim_{h \rightarrow 0} f(x+h) \frac{g(x+h) - g(x)}{h} + g(x) \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \end{aligned}$$

You do: Find $\frac{dy}{dx}$

$$a) y = (\sqrt{x} + 4x)3^x - \frac{14}{x^5}$$

$$b) y = 6(x+3)^2 + \frac{e^x}{x^3}$$

$$c) y = \frac{2x^2 + 1}{x^3 e^x}$$

3.3 Derivatives of Trig Functions

First, some things you must know about trig functions:

$\sec(x) = \frac{1}{\cos(x)}$	$\csc(x) = \frac{1}{\sin(x)}$
$\tan(x) = \frac{\sin(x)}{\cos(x)}$	$\cot(x) = \frac{\cos(x)}{\sin(x)}$

Identities needed for today:

$$\sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b)$$

$$\cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b)$$

Consider $f(x) = \sin(x)$.

Then

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \end{aligned}$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x)$$

$$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\sec(x)) = \sec(x) \tan(x)$$

$$\frac{d}{dx}(\csc(x)) = -\csc(x) \cot(x)$$